Stationary models
MA, AR and ARMA

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Lectures list

1. Stationarity
2. **ARMA models for stationary variables**
3. Seasonality
4. Non-stationarity
5. Non-linearities
6. Multivariate models
7. Structural VAR models
8. Cointegration the Engle and Granger approach
9. Cointegration 2: The Johansen Methodology
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11. Multivariate Nonlinearities in VECM models
1. Last Lecture

2. AR(p) models
   - Autocorrelation of AR(1)
   - Stationarity Conditions
   - Estimation

3. MA models
   - ARMA(p,q)
   - The Box-Jenkins approach

4. Forecasting
Recall: auto-covariance

Definition (autocovariance)
\[ \text{Cov}(X_t, X_{t-k}) \equiv \gamma_k(t) \equiv E[(X_t - \mu)(X_{t-k} - \mu)] \]

Definition (Autocorrelation)
\[ \text{Corr}(X_t, X_{t-k}) \equiv \rho_k(t) \equiv \frac{\text{Cov}(X_t, X_{t-k})}{\text{Var}(X_t)} \]

Proposition
\[ \text{Corr}(X_t, X_{t-0}) = \text{Var}(X_t) \]
\[ \text{Corr}(X_t, X_{t-j}) = \phi^j \] depend on the lage: plot its values at each lag.
Recall: stationarity

The stationarity is an essential property to define a time series process:

**Definition**

A process is said to be **covariance-stationary**, or **weakly stationary**, if its first and second moments are **time invariant**.

\[
egin{align*}
E(Y_t) &= E[Y_{t-1}] = \mu & \forall t \\
\text{Var}(Y_t) &= \gamma_0 < \infty & \forall t \\
\text{Cov}(Y_t, Y_{t-k}) &= \gamma_k & \forall t, \forall k
\end{align*}
\]
Recall: The AR(1)

The AR(1): $Y_t = c + \varphi Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim iid(0, \sigma^2)$
with $|\varphi| < 1$, it can be written as:

$$Y_t = \frac{c}{1 - \varphi} + \sum_{i=0}^{t-1} \varphi^i \varepsilon_{t-i}$$

Its 'moments' do not depend on the time:

- $E(X_t) = \frac{c}{1 - \varphi}$
- $\text{Var}(X_t) = \frac{\sigma^2}{1 - \varphi^2}$
- $\text{Cov}(X_t, X_{t-j}) = \frac{\varphi^j}{1 - \varphi^2} \sigma^2$
- $\text{Corr}(X_t, X_{t-j}) = \varphi^j$
Outline

1 Last Lecture

2 AR(p) models
   • Autocorrelation of AR(1)
   • Stationarity Conditions
   • Estimation

3 MA models
   • ARMA(p,q)
   • The Box-Jenkins approach

4 Forecasting
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A useful plot to understand the dynamic of a process is the autocorrelation function:
Plot the autocorrelation value for different lags.
\[ \phi = 0 \]

\[ \phi = 0.5 \]

\[ \phi = 0.9 \]

\[ \phi = 1 \]
AR(1) with $-1 < \phi < 0$

in the AR(1): \( Y_t = c + \varphi Y_{t-1} + \varepsilon_t \) \( \varepsilon_t \sim iid(0, \sigma^2) \)

with $-1 < \phi < 0$

we have negative autocorrelation.
Definition (AR(p))

\[ y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t \]

- Expectation?
- Variance?
- Auto-covariance?
- Stationary conditions?
Lag operator

Definition (Backshift /Lag operator)

\[ LX_t = X_{t-1} \]

Proposition

See that: \( L^2 X_t = X_{t-2} \)

Proposition (Generalisation)

\[ L^k X_t = X_{t-k} \]
Lag polynomial

We can thus rewrite:

Example (AR(2))

$$X_t = c + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \varepsilon_t$$

$$(1 - \varphi_1 L - \varphi_2 L^2)X_t = c + \varepsilon_t$$

Definition (lag polynomial)

We call lag polynomial: $\Phi(L) = (1 - \varphi_1 L - \varphi_2 L^2 - \ldots - \varphi_p L^p)$

So we write compactly:

Example (AR(2))

$$\Phi(L)X_t = c + \varepsilon_t$$
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4. Forecasting
Definition (Characteristic polynomial)

\[(1 - \varphi_1z - \varphi_2z^2 - \ldots - \phi_pz^p)\]

Stability condition:

**Proposition**

*The AR\((p)\) process is stable if the roots of the lag polynomial lie outside the unit circle.*

**Example (AR\((1)\))**

The AR\((1)\): \(X_t = \varphi X_{t-1} + \varepsilon_t\)

can be written as: \((1 - \varphi L)X_t = \varepsilon_t\)

Solving it gives: \(1 - \varphi x = 0 \Rightarrow x = \frac{1}{\varphi}\)

And finally: \(\left|\frac{1}{\varphi}\right| > 1 \Rightarrow |\varphi| < 1\)
Proof.

1. Write an AR(p) as AR(1)
2. Show conditions for the augmented AR(1)
3. Transpose the result to the AR(p)
Proof.

The AR(p):

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t \]

can be recast as the AR(1) model:

\[ \xi_t = F \xi_{t-1} + \varepsilon_t \]

\[
\begin{bmatrix}
    y_t \\
    y_{t-1} \\
    y_{t-2} \\
    \vdots \\
    y_{t-p+1}
\end{bmatrix}
= 
\begin{bmatrix}
    \phi_1 & \phi_2 & \phi_3 & \ldots & \phi_{p-1} & \phi_p \\
    1 & 0 & 0 & \ldots & 0 & 0 \\
    0 & 1 & 0 & \ldots & 0 & 0 \\
    \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\
    0 & 0 & 0 & \ldots & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    y_{t-1} \\
    y_{t-2} \\
    y_{t-3} \\
    \vdots \\
    y_{t-p}
\end{bmatrix}
+ 
\begin{bmatrix}
    \varepsilon_t \\
    0 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
\]

\[
\begin{cases}
    y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t \\
    y_{t-1} = y_{t-1} \\
    \ldots \\
    y_{t-p+1} = y_{t-p+1}
\end{cases}
\]
Proof.

Starting from the augmented AR(1) notation:

\[ \xi_t = F \xi_{t-1} + \varepsilon_t \]

Similarly as in the simple case, we can write the AR model recursively:

\[ \xi_t = F^t \xi_0 + \varepsilon_t + F \varepsilon_{t-1} + F^2 \varepsilon_{t-2} + \ldots + F^{t-1} \varepsilon_1 + F^t \varepsilon_0 \]

Remember the eigenvalue decomposition: \( F = T \Lambda T^{-1} \)

and the propriety that: \( F^j = T \Lambda^j T^{-1} \)

with

\[ \Lambda^j = \begin{bmatrix} \lambda_1^j & 0 & \ldots & 0 \\ 0 & \lambda_2^j & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_3^j \end{bmatrix} \]

So the AR(1) model is stable if \( |\lambda_i| < 1 \quad \forall i \)
Proof.

So the condition on $F$ is that all $\lambda$ from $|F - \lambda I| = 0$ are $< 1$.
One can show that the eigenvalues of $F$ are:

**Proposition**

$$\lambda^p - \phi_1\lambda^{p-1} - \phi_2\lambda^{p-2} - \ldots - \phi_{p-1}\lambda - \phi_p = 0$$

But the $\lambda$ are the reciprocal of the values $z$ that solve the characteristic polynomial of the AR($p$):

$$(1 - \varphi_1 z - \varphi_2 z^2 - \ldots - \phi_p z^p) = 0$$

So the roots of the polynomial should be $> 1$, or, with complex values, outside the unit circle.
Stationarity conditions

The conditions of roots outside the unit circle lead to:

- **AR(1):** $|\phi| < 1$
- **AR(2):**
  - $\phi_1 + \phi_2 < 1$
  - $\phi_1 - \phi_2 < 1$
  - $|\phi_2| < 1$
Example

Consider the AR(2) model:

\[ Y_t = 0.8 Y_{t-1} + 0.09 Y_{t-2} + \varepsilon_t \]

Its AR(1) representation is:

\[
\begin{bmatrix}
  y_t \\
  y_{t-1}
\end{bmatrix} =
\begin{bmatrix}
  0.8 & 0.09 \\
  1 & 0
\end{bmatrix}
\begin{bmatrix}
  y_{t-1} \\
  y_{t-2}
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_t \\
  0
\end{bmatrix}
\]

Hence its eigenvalues are taken from:

\[
\begin{vmatrix}
  0.8 - \lambda & 0.09 \\
  1 & 0 - \lambda
\end{vmatrix}
= \lambda^2 - 0.8\lambda - 0.09 = 0
\]

And the eigenvalues are smaller than one:

\[
\text{Re(polyroot(c(-0.09, -0.8, 1)))}
\]

\[
[1] -0.1 \quad 0.9
\]
Example

\[ Y_t = 0.8 Y_{t-1} + 0.09 Y_{t-2} + \varepsilon_t \]

Its lag polynomial representation is: \((1 - 0.8L - 0.09L^2)X_t = \varepsilon_t\)

Its characteristic polynomial is hence: \((1 - 0.8x - 0.09x^2) = 0\)

whose solutions lie outside the unit circle:

```r
> Re(polyroot(c(1, -0.8, -0.09)))

[1]  1.111111 -10.000000
```

And it is the inverse of the previous solutions:

```r
> all.equal(sort(1/Re(polyroot(c(1, -0.8, -0.09)))), Re(polyroot(c(-0.09, + -0.8, 1))))

[1] TRUE
```
**Definition**

A process is said to be integrated of order \( d \) if it becomes stationary after being differenced \( d \) times.

**Proposition**

*An AR(p) process with \( k \) unit roots (or eigenvalues) is integrated of order \( k \).*

**Example**

Take the random walk: \( X_t = X_{t-1} + \varepsilon_t \)

Its polynomial is \((1-L)\), and the roots is \( 1 - x = 0 \Rightarrow x = 1 \)

The eigenvalue of the trivial AR(1) is \( 1 - \lambda = 0 \Rightarrow \lambda = 1 \)

So the random walk is integrated of order 1 (or difference stationary).
Integrated process

Take an AR(p):

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t \]

With the lag polynomial:

\[ \Phi(L) X_t = \varepsilon_t \]

If one of its \( p \) (not necessarily distinct) eigenvalues is equal to 1, it can be rewritten:

\[ (1 - L) \Phi'(L) X_t = \varepsilon_t \]

Equivalently:

\[ \Phi'(L) \Delta X_t = \varepsilon_t \]
The AR(p) in detail

Moments of a **stationary** AR(p)
- $E(X_t) = \frac{c}{1-\varphi_1-\varphi_2-\ldots-\varphi_p}$
- $\text{Var}(X_t) = \varphi_1 \gamma_1 + \varphi_2 \gamma_2 + \ldots + \varphi_p \gamma_p + \sigma^2$
- $\text{Cov}(X_t, X_{t-j}) = \varphi_1 \gamma_{j-1} + \varphi_2 \gamma_{j-2} + \ldots + \varphi_p \gamma_{j-p}$

Note that $\gamma_j \equiv \text{Cov}(X_t, X_{t-j})$ so we can rewrite both last equations as:

$$\begin{cases} 
\gamma_0 &= \varphi_1 \gamma_1 + \varphi_2 \gamma_2 + \ldots + \varphi_p \gamma_p + \sigma^2 \\
\gamma_j &= \varphi_1 \gamma_{j-1} + \varphi_2 \gamma_{j-2} + \ldots + \varphi_p \gamma_{j-p} 
\end{cases}$$

They are known under the name of **Yule-Walker** equations.
Yule-Walker equations

Dividing by $\gamma_0$ gives:

\[
\begin{align*}
\rho_0 &= \varphi_1 \rho_1 + \varphi_2 \rho_2 + \cdots + \varphi_p \rho_p + \sigma^2 \\
\rho_j &= \varphi_1 \rho_{j-1} + \varphi_2 \rho_{j-2} + \cdots + \varphi_p \rho_{j-p}
\end{align*}
\]

Example (AR(1))

We saw that:

- $\text{Var}(X_t) = \frac{\sigma^2}{1-\varphi^2}$
- $\text{Cov}(X_t, X_{t-j}) = \frac{\varphi^j}{1-\varphi^2} \sigma^2$
- $\text{Corr}(X_t, X_{t-j}) = \varphi^j$

And we have effectively: $\rho_1 = \varphi \rho_0 = \varphi$ and $\rho_2 = \varphi \rho_1 = \varphi^2$

Utility:

- Determination of autocorrelation function
- Estimation
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4. Forecasting
To estimate a AR(p) model from a sample of T we take $t = T - p$

- **Methods of moments**: estimate sample moments (the $\gamma_i$), and find parameters (the $\phi$) correspondly

- **Unconditional ML**: assume $y_p, \ldots, y_1 \sim \mathcal{N}(0, \sigma^2)$. Need numerical optimisation methods.

- **Conditional Maximum likelihood (=OLS)**: estimate $f(y_T, t_{T-1}, \ldots, y_{p+1}|y_p, \ldots, y_1; \theta)$ and assume $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ and that $y_p, \ldots, y_1$ are given.

What if errors are not normally distributed? **Quasi-maximum likelihood estimator**, is still consistent (in this case) but standard errors need to be corrected.
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Moving average models

Two significations!

- regression model
- Smoothing technique!
MA(1)

Definition (MA(1))

\[ Y_t = c + \varepsilon_t + \theta \varepsilon_{t-1} \]

- \( \mathbb{E}(Y_t) = c \)
- \( \text{Var}(Y_t) = (1 + \theta^2)\sigma^2 \)
- \( \text{Cov}(X_t, X_{t-j}) = \begin{cases} \theta \sigma^2 & \text{if } j = 1 \\ 0 & \text{if } j > 1 \end{cases} \)
- \( \text{Corr}(X_t, X_{t-j}) = \begin{cases} \frac{\theta}{(1+\theta^2)} & \text{if } j = 1 \\ 0 & \text{if } j > 1 \end{cases} \)

Proposition

A MA(1) is stationnary for every \( \theta \)
\( \theta = 0.5 \)

\( \theta = 2 \)

\( \theta = -0.5 \)

\( \theta = -2 \)
\[ \theta = 0.5 \]

\[ \theta = 3 \]

\[ \theta = -0.5 \]

\[ \theta = -3 \]
The MA(q) is given by:

\[ Y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_1 \epsilon_{t-q} \]

- \( E(Y_t) = c \)
- \( \text{Var}(Y_t) = (1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2)\sigma^2 \)
- \( \text{Cov}(X_t, X_{t-j}) = \begin{cases} \sigma^2(\theta_j + \theta_{j+1}\theta_1 + \theta_{j+2}\theta_2 + \ldots + \theta_q\theta_{q-1}) & \text{if } j = 1 \\ 0 & \text{if } j > 1 \end{cases} \)
- \( \text{Corr}(X_t, X_{t-j}) = \begin{cases} \frac{\theta}{(1+\theta^2)} & \text{if } j = 1 \\ 0 & \text{if } j > 1 \end{cases} \)

**Proposition**

A MA(q) is stationary for every sequence \( \{\theta_1, \theta_2, \ldots, \theta_q\} \).
\[ \Theta = c(0.5, 1.5) \]

\[ \Theta = c(-0.5, -1.5) \]

\[ \Theta = c(-0.6, 0.3, -0.5, 0.5) \]

\[ \Theta = c(-0.6, 0.3, -0.5, 0.5, 3, 2, -1) \]
The MA(∞)

Take now the MA(∞):

\[ Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_\infty \varepsilon_\infty = \sum_{j=0}^{\infty} \theta_j \varepsilon_{t-j} \]

**Definition (Absolute summability)**

A sequence is *absolute summable* if \[ \sum_{i=0}^{\infty} |\alpha_i| < 0 \]

**Proposition**

*The MA(∞) is stationary if the coefficients are absolute summable.*
Back to AR(p)

Recall:

**Proposition**

*If the characteristic polynomial of a AR(p) has roots =1, it is not stationary.*

See that:

\[
(1 - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \cdots - \phi_p y_{t-p})y_t = \\
(1 - \alpha_1 L)(1 - \alpha_2 L)\cdots(1 - \alpha_p L)y_t = \epsilon_t
\]

It has a MA(∞) representation if: \( \alpha_1 \neq 1 \):

\[
y_t = \frac{1}{(1-\alpha_1 L)(1-\alpha_2 L)\cdots(1-\alpha_p L)}\epsilon_t
\]

Furthermore, if the \( \alpha_i \) (the eigenvalues of the augmented AR(1)) are smaller than 1, we can write it:

\[
y_t = \sum_{i=0}^{\infty} \beta_i \epsilon_t
\]
Estimation of a MA(1)

We do not observe neither $\varepsilon_t$ nor $\varepsilon_{t-1}$
But if we know $\varepsilon_0$, we know $\varepsilon_1 = Y_t - \theta \varepsilon_0$
So obtain them recursively and minimize the conditional SSR:
$S(\theta) = \sum_{t=1}^{T} (y_t - \varepsilon_{t-1})^2$
This requires numerical optimization and works only if $|\theta| < 1$. 
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4. Forecasting
The ARMA model is a composite of AR and MA:

**Definition (ARMA(p,q))**

\[ X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_{t-1} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta \varepsilon_{t-q} \]

It can be rewritten properly as:

\[ \Phi(L) Y_t = c + \Theta(L) \varepsilon_t \]

**Theorem**

The ARMA(p,q) model is stationary provided the roots of the \( \Phi(L) \) polynomial lie outside the unit circle.

So only the AR part is involved!
Proposition

After q lags, the autocorrelation function follows the pattern of the AR component.

Remember: this is then given by the Yule-Walker equations.
ARIMA(p,d,q)

Now we add a parameter d representing the order of integration (so the I in ARIMA)

**Definition (ARIMA(p,d,q))**

\[
\Phi(L)\Delta^d Y_t = \Theta(L)\varepsilon_t
\]

**Example (Special cases)**

- White noise: ARIMA(0,0,0) \( X_t = \varepsilon_t \)
- Random walk: ARIMA(0,1,0): \( \Delta X_t = \varepsilon_t \Rightarrow X_t = X_{t-1} + \varepsilon_t \)
The MLE estimator has to be found numerically. Provided the errors are normally distributed, the estimator has the usual asymptotical properties:

- Consistent
- Asymptotically efficient
- Normally distributed

If we take into account that the variance had to be estimated, one can rather use the T distribution in small samples.
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The Box-Jenkins approach

1. Transform data to achieve stationarity
2. Identify the model, i.e. the parameters of ARMA(p,d,q)
3. Estimation
4. Diagnostic analysis: test residuals
Step 1

Transformations:
- Log
- Square root
- Differenciation
- Box-Cox transformation: \( Y_t^{(\lambda)} \) \[
\begin{cases} 
\frac{Y_t^\lambda - 1}{\lambda} & \text{for } \lambda \neq 0 \\
\log(Y - t) & \text{for } \lambda = 0
\end{cases}
\]

Is log legitimate?
- Process is: \( y_t e^{\delta t} \) Then \( z_t = \log(y_t) = \delta t \) and remove trend
- Process is \( y_t = y_{t-1} + \varepsilon_t \) Then (by \( \log(1 + x) \cong x \)) \[
\Delta \log(y_t) = \frac{y_t - y_{t-1}}{y_t}
\]
Step 2

Identification of p,q (d should now be 0 after convenient transformation)

Principle of parsimony: prefer small models Recall that

- incorporating variables increases fit ($R^2$) but reduces the degrees of freedom and hence precision of estimation and tests.
- A AR(1) has a MA(∞) representation
- If the MA(q) and AR(p) polynomials have a common root, the ARMA(p,q) is similar to ARMA(p-1,q-1).
- Usual techniques require that the MA polynomial has roots outside the unit circle (i.e. is invertible)
Step 2: identification

How can we determine the parameters $p,q$?

- Look at ACF and PACF with confidence interval
- Use information criteria
  - Akaike Criterion (AIC)
  - Schwarz criterion (BIC)

**Definition (IC)**

\[
\text{AIC}(p) = n \log \hat{\sigma}^2 + 2p
\]

\[
\text{BIC}(p) = n \log \hat{\sigma}^2 + p \log n
\]
Step 3: estimation

Estimate the model...
R function: arima() argument: order=c(p,d,q)
Step 4: diagnostic checks

Test if the residuals are white noise:

1. Autocorrelation
2. Heteroscedasticity
3. Normality
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Series CPI2

Time
CPI2
1985 1995 2005
−0.03 −0.01 0.01
0.5 1.0 1.5 2.0
−0.2 0.0 0.2
Lag
ACF
Series CPI2

Partial ACF
Series CPI2

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> library(forecast)

This is forecast 1.17

> fit <- auto.arima(CPI2, start.p = 1, start.q = 1)
> fit

Series: CPI2
ARIMA(2,0,1)(2,0,2)[12] with zero mean

Coefficients:
                     ar1       ar2       ma1       sar1       sar2       sma1       sma2
       0.2953    -0.2658    -0.9011    0.6021    0.3516    -0.5400    -0.2850
                     s.e.     0.0630    0.0578     0.0304    0.1067    0.1051     0.1286     0.1212

sigma^2 estimated as 4.031e-05: log likelihood = 1146.75
AIC = -2277.46   AICc = -2276.99   BIC = -2247.44

> res <- residuals(fit)
> Box.test(res)

    Box-Pierce test

data:  res
X-squared = 0.0736, df = 1, p-value = 0.7862
Normal Q–Q Plot

Sample Quantiles

Theoretical Quantiles

density.default(x = res)

N = 315   Bandwidth = 0.001481

Density
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Notation (Forecast)
\[ \hat{y}_{t+j} \equiv E_t(y_{t+j}) = E(y_{t+j} | y_t, y_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots) \] is the conditional expectation of \( y_{t+j} \) given the information available at \( t \).

Definition (J-step-ahead forecast error)
\[ e_t(j) \equiv y_{t+j} - \hat{y}_{t+j} \]

Definition (Mean square prediction error)
\[ MSPE \equiv \frac{1}{H} \sum_{i=1}^{H} e_i^2 \]
R implementation

To run this file you will need:

- R Package forecast
- R Package TSA
- Data file AjaySeries2.csv put it in a folder called Datasets in the same level than your .Rnw file
- (Optional) File Sweave.sty which change output style: result is in blue, R commands are smaller. Also in same folder as .Rnw file.