Regime switching models

Structural change and nonlinearities

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Outline

Lectures list

1. Stationarity
2. ARMA models for stationary variables
3. Some extensions of the ARMA model
4. Non-stationarity
5. Seasonality
6. **Non-linearities**
7. Multivariate models
8. Structural VAR models
9. Cointegration the Engle and Granger approach
10. Cointegration 2: The Johansen Methodology
11. Multivariate Nonlinearities in VAR models
12. Multivariate Nonlinearities in VECM models

1. Lectures

2. Structural break
   - Stability tests
   - Tests for a break at known date
   - Estimation of breaks
   - Break at unknown date

3. Regime switching models
   - Threshold autoregressive models
   - Smooth transition regression

Outline

Definition

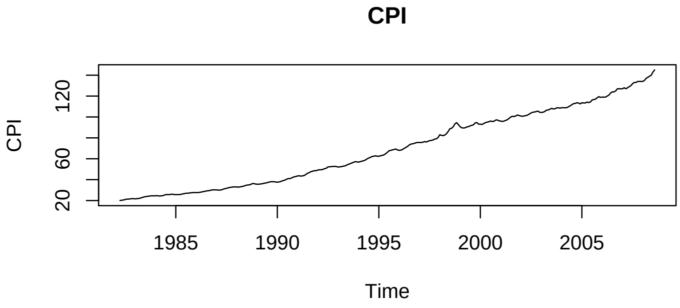
Definition (Structural break/change)

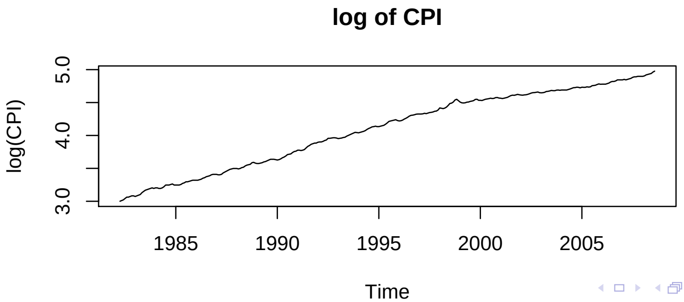
A structural break in an abrupt change in the structure of the modelled relation:

- Univariate model
- Multi-variate (single and multi equation)

It can affect the parameters:

- Slope
- Intercept
- Variance


CPI
CPI
20
60
120
1985
1990
1995
2000
2005
Time


log of CPI
log(CPI)
3.0
4.0
5.0
1985
1990
1995
2000
2005
Time

Outline

Stability tests

Tests of the null of constancy/stability against change at an unspecified date.

- CUSUM test (1975): use recursive residuals
- CUSUM of squares (1975): use squared recursive residuals
- OLS-CUSUM (1992): use OLS residuals
- One-step-ahead prediction error

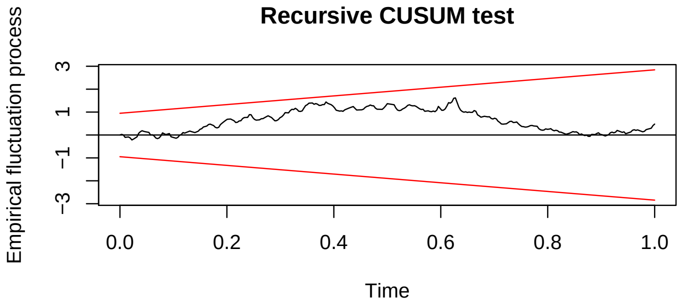
Idea, obtain recursive residual:

$$\hat{\varepsilon}_t = y_t - x_t' \hat{\beta}_{t-1}$$

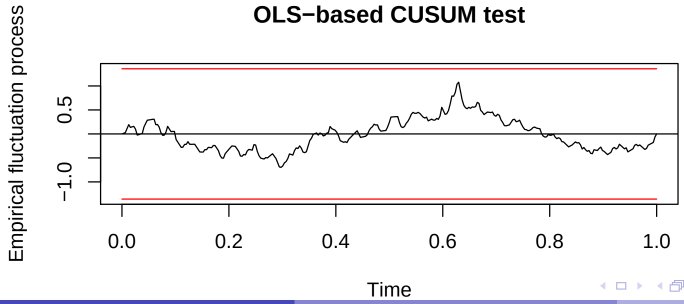
It can be seen as the one-step-ahead prediction error. Normalize it and compute the sum:

$$W_t = \sum w_t$$

If this sum exceeds at the confidence interval: reject H_0



Recursive CUSUM test
Empirical fluctuation process
3
1
−1
−3
0.0
0.2
0.4
0.6
0.8
1.0
Time



OLS−based CUSUM test
Empirical fluctuation process
0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
Time

Problem with stability tests

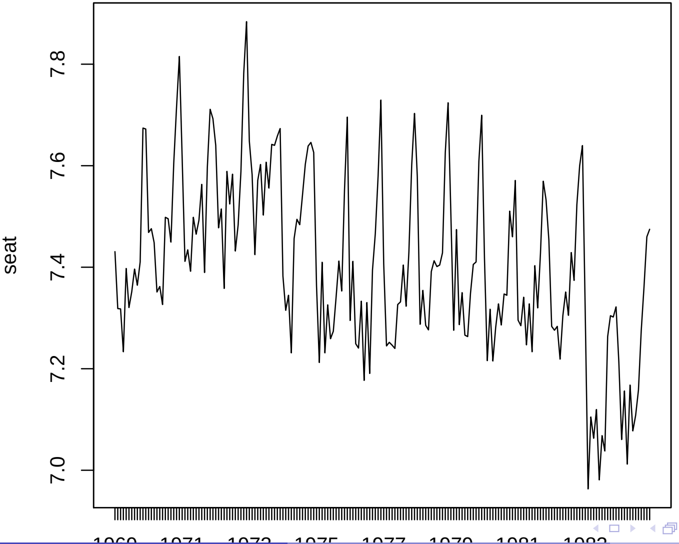
All these tests imply a standardisation: $\frac{\sum e}{\sigma}$
But this variance will increase under H_1!

Under H_1

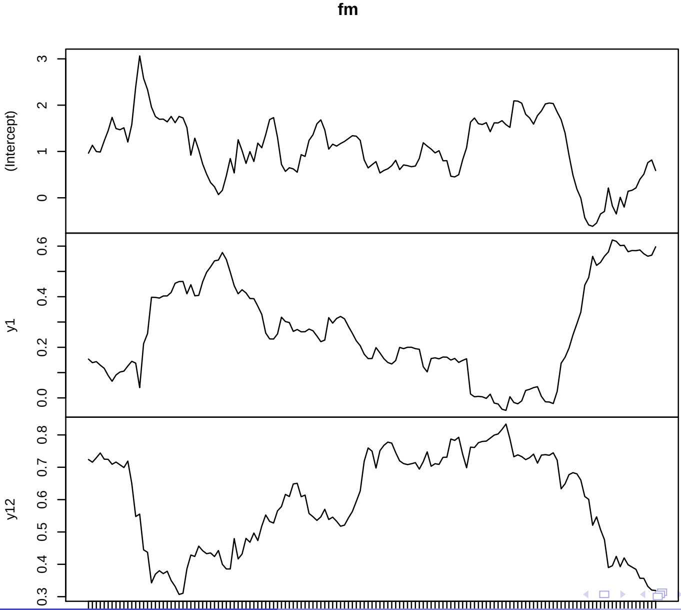
- $\sum e$ increases
- σ increases

So these tests can have low power!

Rolling and recursive regression



Rolling and recursive regression



Recursive regression

Don't move the window, just open it.

Outline

Chow test 1

Chow test: break at time T_1, so we have different coefficients for subsample $[1 : T_1], [T_1 + 1 : T]$

Compare the sum of squares (SSR) of the (restricted) model with same parameters and with different ones:

$$\frac{(SSR_C - (SSR_1 + SSR_2))/(k)}{(SSR_1 + SSR_2)/(T - 2k)} \quad \sim F_{k,T-2k}$$

with k = number of restrictions (and hence 2k is the total number of parameters in the unrestricted model).

Chow test 2

How to do when second sample has $n_2 < k$ (k variables)?

Make a prediction test: estimate X_{n1} and forecast X_{n2}. Compare results.

Subset break at known date

It is also possible to allow for only some coefficients to change.

- Unrestricted model: all coefficients are different
- Restricted model: the subset coefficients are not different

Then apply last formula.

Unequal variance

Notice:

- The tests are for change on the slope and intercept parameters, not the variance.
- We made implicitly assumption of equal variance!

So the restricted model (coefficients are the same) there is heteroskedasticity.

There are some tests which take into account this heteroskedasticity

Break at unknown date

The endogeneity criticism:
Choosing a break date is made from the data, so the choice is not exogenous, as the break is correlated to the data. If the date is not known

a priori, we may wish to test if there is a break, and at which time it occured. There are two questions:

- Was there a break?
- If yes, at which time?

Outline

Multiple breakpoints

Estimation is in two steps (conditional OLS):

1. Minimize (usual OLS estimator) the SSR conditional on the m breaks.
2. Upon all SSR computed, find the m values that lead to the min of the SSR

Bai and Perron (1998) generalise the framework to m breaks (m+1 regimes):

$$y_t = x_t'\beta + z_{t1}'\delta_1 + z_{t2}'\delta_2 + \ldots + z_{tm+1}'\delta_1 + \varepsilon_t$$

Which method?

- Grid search $O(T^m)$
- Algorithm of Bai and Perron (2003)
- Sequential search

Estimates of the breaks

Under the assumption:

- Distance betwen each break increases at rate T
- Short memory of the process (ergodicity)

Proposition

The estimates of the breakpoints are independent.

Proposition

The breakpoint estimates converge at rate T

Estimates of the usual slope estimators

Proposition

The usual slope parameter converge at rate $\sqrt{T}$

Proposition

As the breakpoint converge at rate T, they can be considered as given and usual inference is made on the $\hat{\beta}_i$

Problematic assumptions

The usual assumption is:

$$\frac{T_1}{T} = \lambda$$

Why? If T_1 is taken as fixed, then $\lambda \xrightarrow{\infty} 0$

Economic interpretation?

Inference on the breaks

Perron (1997) shows how to obtain the limiting distribution. So confidence intervals can be build.

This is implemented in package *strucchange* as function *confint()*.

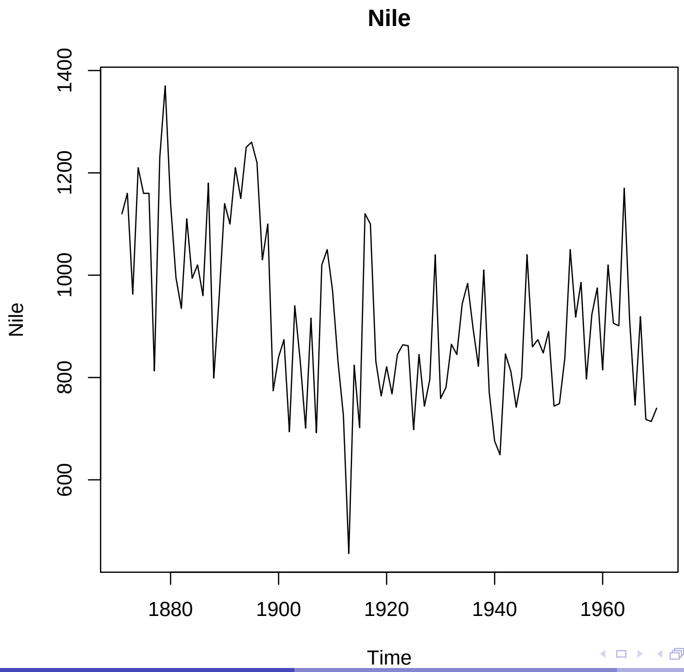
Number of breaks

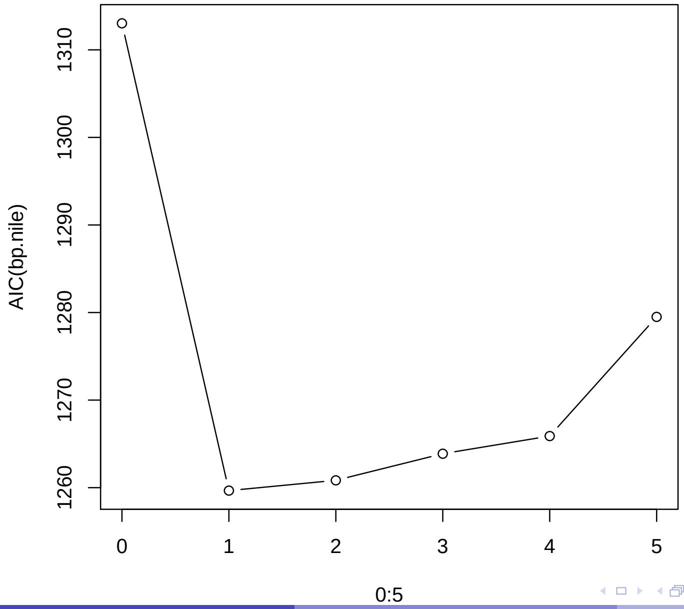
With regime switching models, the presence of a break can't be tested as usually:

$H_0 : \quad T_1 = 0$ does not make sense!

Hence two methods are used:

- Information criterion (AIC, BIC, modified versions)
- Testing procedure


Nile
1400
1200
1000
800
600
Nile
1880
1900
1920
1940
1960
Time


AIC(bp.nile)
1260
1270
1280
1290
1300
1310
0
1
2
3
4
5
0:5

Outline

Break at unknown date

The problem of the unidentified parameter under the null.

If you test:

- H_0: no break
- H_1. break at unknown date

There is a parameter that does not exist under H_0!

Conventional statistical theory cannot be applied to obtain the (asymptotic) distribution of the test statistics. Instead, the test-statistics tend to have a **nonstandard distribution**, *for which an analytical expression is often not available.*

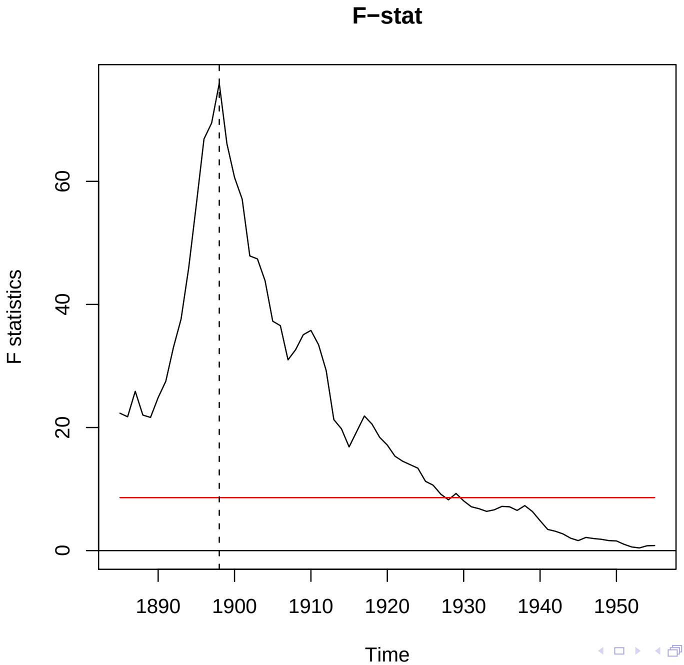
Solution to the unidentification problem

Evalue your test (LR, LM, Wald) for each value and use:

- Supremum
- Average
- Exponential

Actually, not for each value: exclude $a\%$ in the beginning and end of the series. Often, $a = 15\%$.

If take too low: power decreases.


F−stat
F statistics
0
20
40
60
1890
1900
1910
1920
1930
1940
1950
Time

```
> summary(breakpoints(fs.nile))

         Optimal 2-segment partition:

Call:
breakpoints.Fstats(obj = fs.nile)

Breakpoints at observation number:
28

Corresponding to breakdates:
1898

RSS: 1597457
```

Test at unknown date

There are tests:

- No break against one break at unknown date
- No break against mutliple breaks at unknown date
- l breaks against l+1 breaks (implemented in strucchange?)

I(1) variables

The previous tests are based on I(0) variables.

	I(0)	I(1)
No structural change		
Structural change		

I(1) with known break

Perron (1988) test with known date

- H_0: one time jump in I(1)
- H_1: one time change in the trend/intercept in I(0)

Application to Nelson-Plosser (1982) data: most of the series do not contain any longer a unit root

Break under a RW and an AR

Recall the different interpretation of the const/trend under a RW or a AR!
We need different dummy to model the same change under RW or AR.
Change in level:

- RW: $y_t = y_{t-1} + \mu D_P + \varepsilon_t$
- AR: $y_t = y_{t-1} + D_L + \varepsilon_t$

where: $D_P \begin{cases} 1 & \text{if } t = t_1 + 1 \\ 0 & \text{else} \end{cases}$ $D_L \begin{cases} 1 & \text{if } t > t_1 \\ 0 & \text{else} \end{cases}$

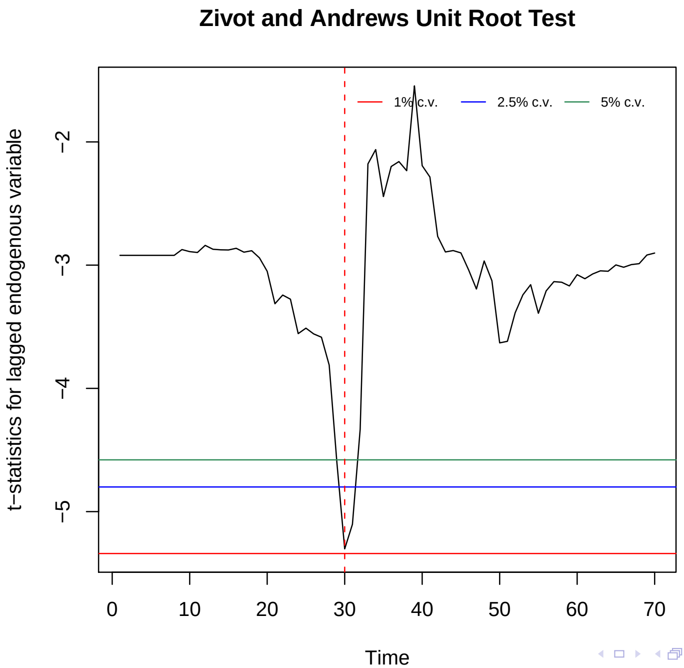
I(1) with unknown break

Zivot and Andrews (1992) test:

- $H_0 : y_t = Y_{t-1} + \varepsilon_t$ RW with drift and **without** break.
- H_A : Trend stationary model with break in slope/trend/both.

Break date is unknown: compute all p-values and take the minimum.

Nelson-Plosser data: less evidence for rejection.


Zivot and Andrews Unit Root Test
1% c.v.
2.5% c.v.
5% c.v.
t−statistics for lagged endogenous variable
−2
−3
−4
−5
0
10
20
30
40
50
60
70
Time

Composite hypothesis

We can also try if the variable is I(1) and then I(0) or opposite.
Kim (2000) test:

- H_0: series is I(0)
- H_1: switch to I(1) to I(0) or vice-versa

Leybourne et al. (2003) test:

- H_0: series is I(1)
- H_1: switch to I(1) to I(0) or vice-versa

But what result should we have if the variable is I(1)/I(0) on the whole sample?

Outline

Outline

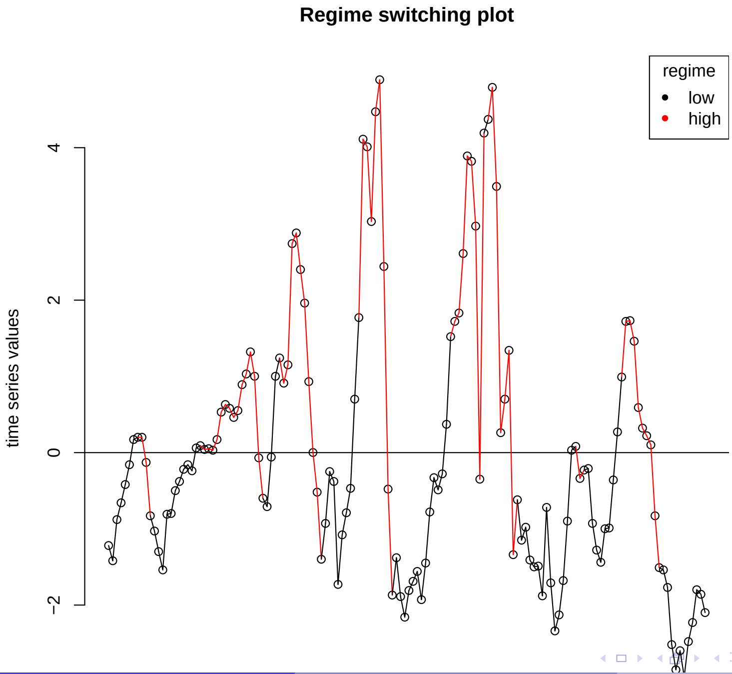
TAR framework

Self-exciting Threshold Autoregressive model

SETAR with m regimes (m-1 thresholds)

$$y_t = \begin{cases} \mu^1 + \rho_1^1 y_{t-1} + \ldots + \rho_{p1}^1 y_{t-p1} + \varepsilon_t & \text{if} \quad x_{t-d} \geq \theta_{m-1} \\ \mu^2 + \rho_1^2 y_{t-1} + \ldots + \rho_{p2}^2 y_{t-p2} + \varepsilon_t & \text{if} \quad \theta_{m-1} \geq x_{t-d} \geq \theta_{m-2} \\ \ldots & \text{if} \quad \theta_{\ldots} \geq x_{t-d} \geq \theta_{\ldots} \\ \mu^m + \rho_1^m y_{t-1} + \ldots + \rho_{pm}^m y_{t-pm} + \varepsilon_t & \text{if} \quad \theta_1 \geq x_{t-d} \end{cases}$$

- x_{t-d} is the transition variable (time for structural break)
- d is the delay of the transition variable


Regime switching plot
regime
low
high
time series values
4
2
0
−2

Conditions for stationarity

The SETAR framework allow an interesting idea: be locally non-stationary (in the corridor) but globally stationary.
Conditions for the restrictive cases: d=p=1

- $\rho^{(l)} < 1, \rho^{(u)} < 1,$ and $\rho^{(l)}\rho^{(u)} < 1$
- $\rho^{(l)} < 1, \rho^{(u)} = 1,$ and $\mu^{(u)} < 0$
- $\rho^{(u)} < 1, \rho^{(l)} = 1,$ and $\mu^{(l)} > 0$
- $\rho^{(u)} = \rho^{(l)} = 1,$ and $\mu^{(u)} < 0 < \mu^{(l)}$
- $\rho^{(u)}\rho^{(l)} = 1, \rho^{(l)} < 0,$ and $\mu^{(u)} + \rho^{(u)}\mu^{(l)}$

Stationarity with unit roots

A TAR model can be globally stationary even if each regime has a unit root!

TAR specifications

We will see three specifications of TAR models based on Balke and Fomby (1997)

- Equilibrium-TAR
- Band-TAR
- RD-TAR

All these models are with p=d=1 and $r^{(u)} = r^{(u)}$

First condition can't be easily relaxed, second can be.

Equilibrium-TAR

$$y_t = \begin{cases} \rho y_{t-1} + \varepsilon_t & \text{if } y_{t-1} > r \\ y_{t-1} + \varepsilon_t & \text{if } -r < y_{t-1} < r \\ \rho y_{t-1} + \varepsilon_t & \text{if } y_{t-1} < -r \end{cases}$$

Adjustment to the "equilibrium " (=0 as no constant in the corridor?)

Band-TAR

$$y_t = \begin{cases} r(1-\rho) + \rho y_{t-1} + \varepsilon_t & \text{if } y_{t-1} > r \\ y_{t-1} + \varepsilon_t & \text{if } -r < y_{t-1} < r \\ -r(1-\rho) + \rho y_{t-1} + \varepsilon_t & \text{if } y_{t-1} < -r \end{cases}$$

Remember that in an AR(1): $y_t = c + \rho y_{t-1} + \varepsilon_t, \qquad \mathrm{E}[y_t] = \frac{c}{1-\rho}$
So adjustment to the band only

Returning drift-TAR

$$y_t = \begin{cases} -\mu + y_{t-1} + \varepsilon_t & \text{if } y_{t-1} > r \\ y_{t-1} + \varepsilon_t & \text{if } -r < y_{t-1} < r \\ \mu + y_{t-1} + \varepsilon_t & \text{if } y_{t-1} < -r \end{cases}$$

Comparisons of the models

Band-TAR is more persistent (less adjustment) than the EQ-TAR.
Define ratio: $\frac{r^2}{\sigma_m^2}$
It is a mesure of persistence: *expected hitting time of reaching the thresholds starting from zero*.

- r is big (ratio is big): need much time/big deviations to reach the adjustment regimes
- σ_m^2 is small (ratio is big): don't go often to the adjustment regimes.

Momentum TAR

Transition variable is Δy_{t-d}

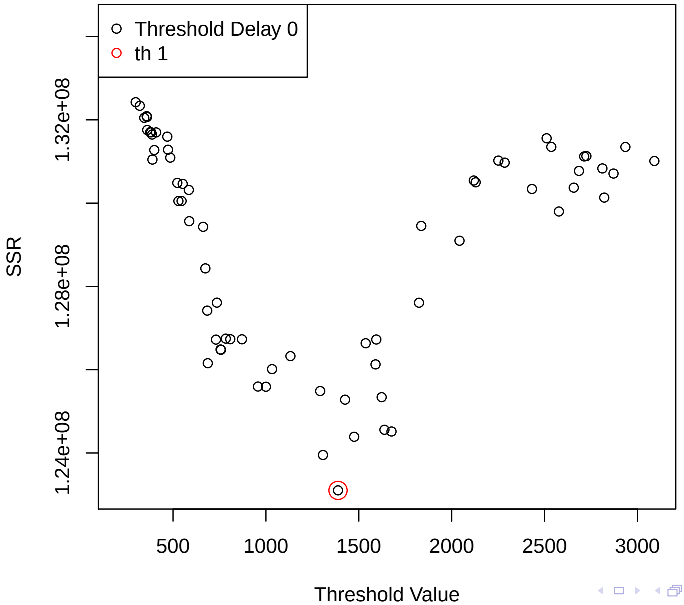
Estimation

Same methods as structural break: conditionnal OLS for:

- Threshold value
- Threshold delay value

Need grid search, can use methods from Bai and Perron (1998)

Results of the grid search



Inference

Chan 1993:

Proposition

The distribution of the threshold parameter is a "compound poisson process" with nuisance parameters

Tests for SETAR

Hansen (1998): AR against SETAR(l)

- AR() vs SETAR(1) or SETAR(2)
- SETAR(1) vs SETAR(2)

Caner and Hansen (2001):

- RW vs RW-SETAR(1)
- RW against M-SETAR (1)
- RW against partial M-SETAR (1): $H_1 : \quad \phi_1$ or $\phi_1 < 0$
- Partial vs total M-SETAR(1)

Both tests are with bootstrap distributions.

Tests for SETAR

Tests suggested: unit-root against SETAR.

- Enders and Granger (1998), RW against SETAR(1): $H_0: \quad \phi_1 = \phi_2 = 0$ (F-stat) for TAR and M-TAR and if rejected check if $\rho_1 = \rho_2$
- Seo (2008): RW against SETAR(1): $H_0: \quad \phi_1 = \phi_2 = 0$ sup-wald stat, with bootstrap distribution
- Shin (2006): RW against SETAR(2): $H_0: \quad \phi_1 = \phi_3 = 0$ (outer coefficients, RW in inner-band assumed),

All these tests have $H_0: \quad \phi_1 = \phi_2 = 0$ and $H_1: \phi_1 < 0 \quad \phi_2 < 0$
So don't test whether $\phi_1 = \phi_2 (\Leftrightarrow$ threshold effect).

To discuss

- Estimation
- Distributions
- Testing approaches
- Choice of the lags
- Choice of the threshold variable
- I(1) and I(0)

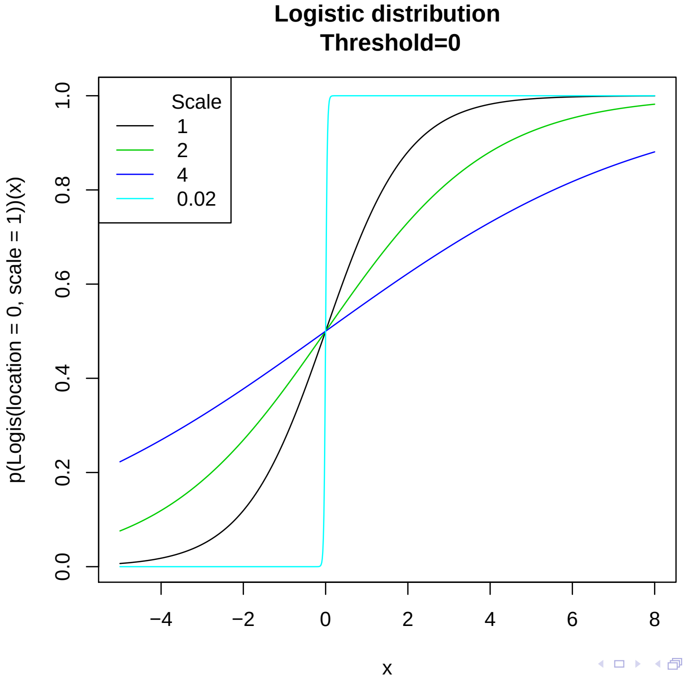
Outline

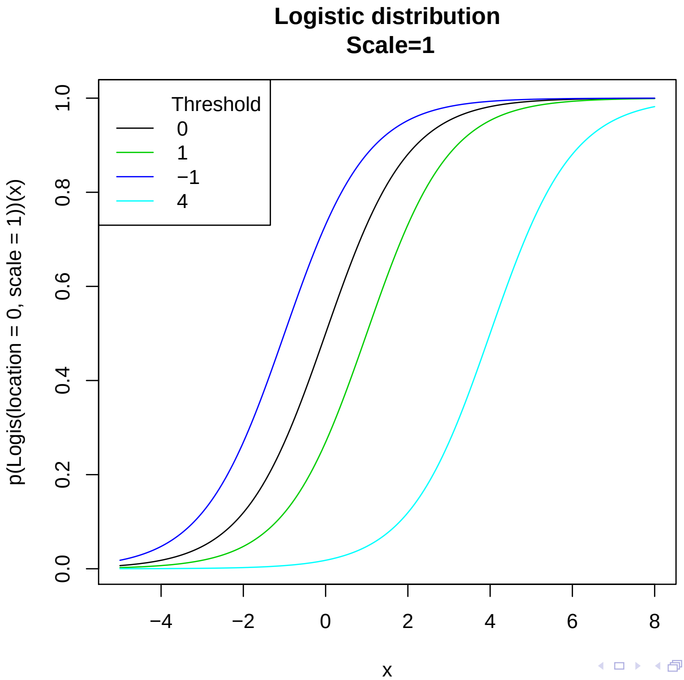
STARs

$$y_t = \mathbf{X_t}\gamma^{(1)}G(z_t, \zeta, c) + \mathbf{X_t}\gamma^{(1)}(1 - G(z_t, \zeta, c)) + \sigma^{(j)}\epsilon_t$$

With G the transition function:

$$G(z_t, \zeta, c)x = \frac{1}{(1 + exp(-\zeta(z_t - c))}\zeta > 0$$


Logistic distribution
Threshold=0
Scale
1
2
4
0.02
p(Logis(location = 0, scale = 1))(x)
0.0
0.2
0.4
0.6
0.8
1.0
−4
−2
0
2
4
6
8
x


Logistic distribution
Scale=1
Threshold
0
1
−1
4
p(Logis(location = 0, scale = 1))(x)
0.0
0.2
0.4
0.6
0.8
1.0
−4
−2
0
2
4
6
8
x

Transition Functions Three basic transition functions and the name of resulting models are:

- first order logistic function - results in Logistic STAR ("'LSTAR"') model:

$$G(z_t, \zeta, c)x = \frac{1}{(1 + exp(-\zeta(z_t - c))}\zeta > 0$$

- exponential function - results in Exponential STAR ("'ESTAR"') model:

$$G(z_t, \zeta, c)x = \frac{1}{1 - exp(-\zeta(z_t - c))}\zeta > 0$$

- second order logistic function:

$$G(z_t, \zeta, c)x = \frac{1}{(1 + exp(-\zeta(z_t - c_1)(z_t - c_2))}\zeta > 0$$

Testing for STAR

The null of no star can be:

- $\phi_A = \phi_B$
- Scale parameter=0 (then $G() = 0.5 \forall y_t$

But in both cases unidentified parameters remain!

Luukkonen, Saikkonen and Tersvirta (1988) find a reparametrisation whith no unidentified parameters and use a LM test.

Smooth stuctural break

Has been applied to structural brek models with smooth change.

Running this sweave+beamer file

To run this Rnw file you will need:

- Package strucchange, urca, distr
- Working version of package tsDyn (here: revision
- Image RegimeChangesin Datasets
- (Optional) File Sweave.sty which change output style: result is in blue, R commands are smaller. Should be in same folder as .Rnw file.